

Hemodynamics with total energy

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Abstract- The arterial vessel is considered together with the blood as a whole system. Taking all the potential energies into account, the system is subjected not only the pressure gradient force but also the elastic forces arising from the longitudinal stretching and the circumferential stretching. It is the longitudinal stretching force that takes the whole arterial system together, and makes the energy traveling from one Windkessel unit to the adjacent Windkessel unit.

Keywords – arterial wall, longitudinal stretching, radial movement

Circulatory system is composed of a heart and many elastic vessels with blood circulating in it. The most dramatic structure which quite different from the ordinary fluid transportation system is the 180-degree of bending at the arch of aorta near the ventricular ejection. This bending causes the blood convert great amount of kinetic energy into the potential energy of the elastic wall. In 1966, Shepard et al. [1] and Milnor et al. [2] pointed that the kinetic power is only 2% to 7% of the total ventricular output under basal resting conditions. That is, the power associated with the potential is of more than 90%.

Arteries in the body are naturally under a condition of longitudinal tension. The average value of the relative shortening in terms of in vivo length is ranged from 21% to 42%. In 1964, Patel et al. [3] has found that, in situ, the tethering of the arteries in the longitudinal direction is greater than in the circumferential directions at usual physiologic pressures. This discovery reminded us the vibration of a string. Maintaining a steady transverse vibration in a long taunt string consumes quite small amount of power, whereas, it is so hard to transport a longitudinal fluid vibration to a long distance with pure pressure gradient force.

The effect of longitudinal stretching in blood flow has seldom been mentioned. The main reason is that most of the traditional hemodynamic studies focused on the blood flow system alone. The wall was taken as a boundary of the blood system and assumed fixed at first. Its movement was considered only as a correction term. The important energy associated with the transverse vibration was then totally neglected at the first place.

As a matter of fact, more than 90% of the energy in arterial system is stored in the pressure energy of the fluid as well as in the longitudinal stretching potential energy, the circumferential stretching potential energy and the transverse kinetic energy of the vessel. As the fluid and the elastic vessel contacting with each other, there is strong energy transfer between them. The contact force is too complicated

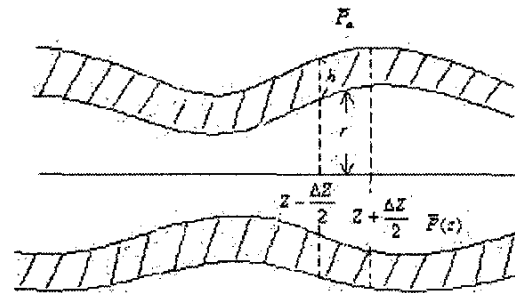


Fig.1 (a): Inside the dotted line is a segment of the artery between the axial positions $Z - \Delta Z/2$ and $Z + \Delta Z/2$. The inner fluid pressure is $\bar{P}(z)$, and the outside atmosphere pressure is $\bar{P}_a$. The vessel has a local inner radius $r(z)$ and wall thickness h .

to be described and too large to be neglected. The best way to solve these two strong interaction systems without missing the important part of the energy is to consider them together as a one system. Thus, all the complicated contact forces become internal forces. By Newton's third law, the action and the reaction forces will cancel each other and need not be considered in the equation of motion of the total system. Figure 1(a) shows a segment of the arterial system. The internal fluid pressure is $\bar{P}$ and the external atmosphere pressure is $\bar{P}_a$. The inner radius of the vessel r is varied with position z due to the transverse vibration of the arterial system. The thickness of the wall is h . Figure 1(b) is a basic element to be considered. It is a system composed of a thin layer of vessel wall and a wedged-shaped of fluid with azimuth angle $d\theta$.

Contacting with the neighboring outside systems, there are five surfaces at the vessel and four surfaces at the fluid. (The contact surface between the fluid and the wall is not expressed since the forces acting on this surface are internal forces.)

The equations of motion of this combined system are related only to the forces acting by its surrounding systems.

On surfaces 1 and 2, there are pressure forces $A_1 \bar{P}_1$ and $A_2 \bar{P}_2$ acting by the outside adjacent fluid in opposite axial

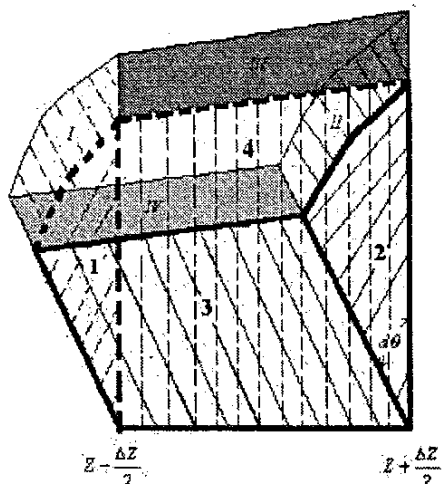


Fig.1 (b): A small element of azimuth angle $d\theta$. It is composed of a thin layer of vessel and a wedge-shaped fluid. The element is contacting with the surrounding via five surfaces at the vessel and four surfaces at the fluid.

direction, and the total effect is an area gradient force F_A accompanying with the pressure gradient force F_p .

On surfaces 3 and 4, the pressure forces of the outside fluid will have a net radial component acting on the whole system.

The elastic vessel is subjected a longitudinal tension per unit circumferential length τ . On surfaces I and II, longitudinal stretching forces $\tau_1 d\theta$ and $\tau_2 d\theta$ are acting on the system by their adjacent vessels. They will contribute an effective tension forces F_z in the axial direction and F_r in the radial direction.

Under circumferential stretching, the restoring forces acting by the adjacent walls are acting on surfaces III and IV. The sum of the two forces F_c is in the radial direction.

On the upper surface of the wall, there is a normal force F_a acting by the outside atmosphere.

We may take all those forces together to the Newton's equation of motion of the unit system shown in figure 1(b). We [4, 5] have derived that the radial component will lead a pressure wave equation with high frequency phase velocity related to the longitudinal tension. The axial component will lead a flow equation driven not only by the pressure gradient force F_p but also the area gradient force F_A and longitudinal stretching force F_z .

From the above discussion, we conclude that the main artery is an energy reservoir. The power delivered from the heart is transported to the whole arterial system via the longitudinal stretching force. When the blood vessel conducting a steady transverse vibration, most of the energy

is transferring between the pressure energy of the fluid and the elastic energies of the vessel. Only small amount of energy is dissipated in the transverse resistance force and the fluid viscous force.

The Windkessel model has considered the circumferential stretching energy. However, it is the longitudinal stretching energy that takes the whole arterial system together, and makes the energy traveling from one Windkessel unit to the adjacent Windkessel unit.

Thus from total energy concern, we suggest further research work should take more weight on the longitudinal stretching. We would also suggest that we should take the whole circulatory system together and study their distributed oscillating modes.

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